Modern Portfolio Theory and the Prudent Investor Act

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Editor's Note: Modern Portfolio Theory has become a customary tool used by investment professionals and, as such, constitutes an industry standard that prudent fiduciaries cannot ignore. Further, the Prudent Investor Rule and Modern Portfolio Theory are inextricably intertwined. We have elected to publish four articles in consecutive editions of ACTEC Journal *in order to provide our readership with an understanding of Modern Portfolio Theory, demonstrate the necessity of applying this theoretical construct in accordance with the Prudent Investor Rule and apply this theory to other pertinent issues surrounding the administration and litigation of trust portfolios. Sequential publication eliminates the need to redevelop Modern Portfolio Theory and other concepts in each article.* ACTEC Journal *readers will have the option of reviewing earlier articles to clarify any points of interest in subsequent articles.*

This first article will provide a foundation for understanding the underpinnings of Modern Portfolio Theory and how it should be applied under the Prudent Investor Rule. The articles to follow in this series are: "Using a Trust's Investment Policy Statement to Develop the Portfolio's Appropriate Risk Level," "Computing Market Adjusted Damages in Fiduciary Surcharge Cases Using Modern Portfolio Theory," and "The Appropriate Withdrawal Rate: Comparing a Total Return Trust to a Principal and Income Trust."

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I. Introduction

The Prudent Investor Rule (Rule) gives personal representatives and fiduciaries both the authority *and* the requirement to consult the well-established principles, strategies, and tools of Modern Portfolio Theory (MPT). The Rule requires the test of fiduciary conduct be undertaken from a portfolio formulation perspective without considering the subsequent performance of the portfolio. Formulation means assembling and maintaining a portfolio of assets with a risk tolerance suitable for the purposes, term, distribution requirements, and other conditions of the trust. MPT guides the fiduciary in constructing and managing a portfolio that provides the highest expected return for that risk tolerance. The responsible fiduciary must do no less. Further, MPT focuses on the whole portfolio, not the individual assets. Finally, MPT requires that the fiduciary develop defensible expectations of return and risk for all potential assets. These principles of MPT, therefore, play a major role in evaluating the actions of fiduciaries as prescribed by the Rule. Section II of this article establishes the basis in trust law for MPT. Sections III and IV explain the underpinnings of MPT and demonstrate how the fiduciary, in compliance with the Rule, should manage the asset allocation of a portfolio. Further, we allude to the potential damages or surcharges associated with the inefficient allocation of assets and/or mistaken determination of an appropriate portfolio risk level.[1] Section V concludes with a summary of fiduciary conduct under MPT and the Rule.

II. Modern Portfolio Theory Determines Prudent Fiduciary Investment Conduct

Some form of the Uniform Prudent Investor Act[2] (Act) has been passed in most every jurisdiction in the United States. The Act is based on the Prudent Investor Rule as more thoroughly developed in the Restatement (Third) of Trusts, Prudent Investor Rule (1992).[3] In turn, the Rule is based upon a large body of academic work that has come to be known as MPT. Professor Harry M. Markowitz first espoused the principles of MPT based upon research he began in 1950 while a Ph.D. candidate in economics at the University of Chicago. Forty years later, he was awarded the Nobel Prize in Economic Sciences for his part in developing MPT.

Though the words, "Modern Portfolio Theory," are not used in the Act, the associated commentary and the Restatement identify MPT as the source of investment guidance for fiduciaries. For example in its introduction, the Restatement says:

> "…criticism [under the old law] is found in writings that have collectively and loosely come to be called ***modern portfolio theory*** [emphasis added]." Further: "What has come to be called "***modern portfolio theory*** [emphasis added]" offers an instructive conceptual framework for understanding and attempting to cope with non-market risk."[4]

Furthermore the Act's Prefatory Note says:

> the Act "…undertakes trust investment law in recognition of the alterations that have occurred in investment practice. These changes have occurred under the influence of a large and broadly accepted body of empirical and theoretical knowledge about the behavior of capital markets, often described as ***modern portfolio theory*** [emphasis added].

Finally one commentator notes:

> "Both the Restatement and the [Act] specifically accept modern portfolio theory…The comments to the Restatement and the Reporter's Notes contain an extensive discussion of the elements of modern portfolio theory, ***including the relationship of risk and return*** [emphasis added]."[5]

The language of Modern Portfolio Theory is found throughout the Act. For example correlation (the impact of each asset on others in the portfolio) is central to MPT. Both the Act and the Restatement address this important principle:

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[1] Assessing damages are discussed in greater detail in the third article in this series, "Computing Market Adjusted Damages in Fiduciary Surcharge Cases using Modern Portfolio Theory."

[2] Sometimes, the Act is referred to as UPIA, not to be confused with the recently enacted Uniform Principal and Income Accounting Act. Both acts share some related concepts. Specifically, total return concepts under the Uniform Principal and Income Accounting Act were enacted to parallel concepts in MPT incorporated into the Act.

[3] Hereinafter, "Restatement."

[4] *Restatement (Third) of Trusts: Prudent Investor Act*, §227, cmt. e, page 19 (1992).

[5] Martin D. Begleiter, *Does the Prudent Investor Need the Uniform Prudent Investor Act?* Vol. 51, Main Law Review, 1999.

> "Among circumstances that a trustee shall consider…the role that each investment or course of action plays within the overall trust portfolio… ."[6]
>
> "…requires the exercise of reasonable care, skill, and caution, and is to be applied to investments not in isolation but in the context of the trust portfolio…."[7]
>
> "…effective diversification depends not only on the number of assets in a trust portfolio but also on the ways and degrees in which their responses to economic events tend to cancel or neutralize one another."[8]
>
> "…an otherwise dubious, volatile investment can make a major contribution to risk management if the shifts in its returns tend not to correlate with the movements of other investments in the portfolio."[9]
>
> "…as a result of the tendency of the value fluctuations of different assets to offset one another, a portfolio's risk is less than the weighted average of the risk of its individual holdings."[10]

The Rule (and, therefore, the Act)[11] and MPT are inextricably intertwined. Despite the self-evident case for this relationship, the following is heard periodically: "As a fiduciary do I have to employ the concepts of MPT—after all, is not my reasonable business judgment sufficient?" In our concluding section we address the difficulty of relying solely on business judgment. The relationship between the Act and MPT implies that fiduciaries ignoring the tenets of MPT are potentially inconsistent with the Act and the Rule and may put themselves at risk.

III. Modern Portfolio Theory

A. **The Basics of Modern Portfolio Theory.** While MPT is usually discussed by academics on a highly theoretical plane, the concepts are not obscure. The primary rule of MPT is the following dictum: For every level of expected risk, a portfolio can be constructed to achieve the highest expected return or, alternatively, for any given level of expected return, a portfolio can be constructed to have the lowest expected risk. Portfolios having these characteristics lie on or quite close to the Efficient Frontier. Under MPT an Efficient Frontier is constructed in expected risk/return space, where return is the expected return of the portfolio and risk is measured by the standard deviation or volatility of the portfolio. The construction of an Efficient Frontier will be fully developed below.

B. **Forecasting Returns.** The expected return of any portfolio can be forecast in a relatively straightforward manner: it is the weighted average of the expected returns of the assets in the portfolio, with the weights being the proportions of the individual assets' market values relative to the market value of the total portfolio.

C. **Forecasting Risk.** The risk (standard deviation) of a portfolio, however, is not the weighted average of the expected standard deviations of the constituent assets. Risk goes beyond the individual standard deviations to encompass the inter-asset correlations or how each asset moves with every other asset in the portfolio.[12] Because the portfolio is the appropriate level of analysis under the Rule, estimating the expected returns, standard deviations, and correlations for every asset in the portfolio are all reasonable duties of the fiduciary.

D. **The Portfolio Effect.** The importance of asset return correlations is probably the most practical contribution of MPT and constitutes the portfolio effect. This effect means that the fiduciary cannot make portfolio decisions by viewing the risk and return characteristics of one asset or asset class in isolation but must take into account how this asset's return correlates with all the other assets in the portfolio.

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[6] Uniform Prudent Investor Act, §2.(c)(4).

[7] *Restatement (Third) of Trusts, Prudent Investor Rule*, §227(a) (1992).

[8] *Restatement (Third) of Trusts, Prudent Investor Rule*, §227, cmt. g, pages 26, 27 (1992).

[9] *Id.*

[10] *Id.*

[11] Practitioners will note the Act applies only to trusts. In some states, application of prudent investor laws may be much broader. Therefore, each practitioner needs to analyze related laws in her or his own jurisdiction and to understand possible subtle nuances. Moreover, in its Prefatory Note, the Act contemplates its potential application to "Other Fiduciary Relationships." *E.g.*, See: Fla. Stat. §518.11 (Florida's Prudent Investor Act), by its own language, applies to trusts, guardianships, and probate estates. However, Fla. Stat. §518.10 applies Fla. Stat. §518.11 to any fiduciary; defined as any "person, whether individual or corporate, who …has the responsibility for acquisition, investment, reinvestment, exchange, retention, sale, or management of money or property of another." Perhaps in a case of over exuberance, Florida law applies the same standard to holders of durable powers of attorney through a tertiary application; *i.e.*, Fla. Stat. §709.08(8) (Florida's Durable Power of Attorney Statute) incorporates the same standard applicable to trustees under Fla. Stat. §737.302 (Florida's Trust Code) which, in turn, incorporates Fla. Stat. §518.11.

[12] Correlation is an index, measured from -1 to +1, that indicates the degree to which returns from two assets move together (approaching +1) or in opposite directions (approaching -1).

Chart III.1
Return Patterns with Negative Correlation and High Portfolio Effect

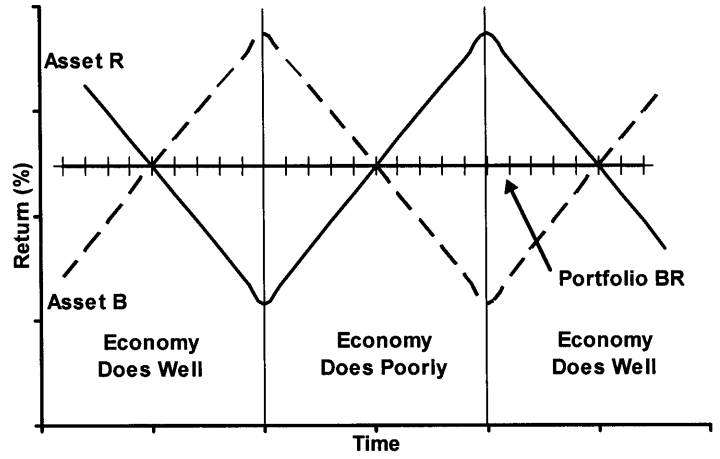
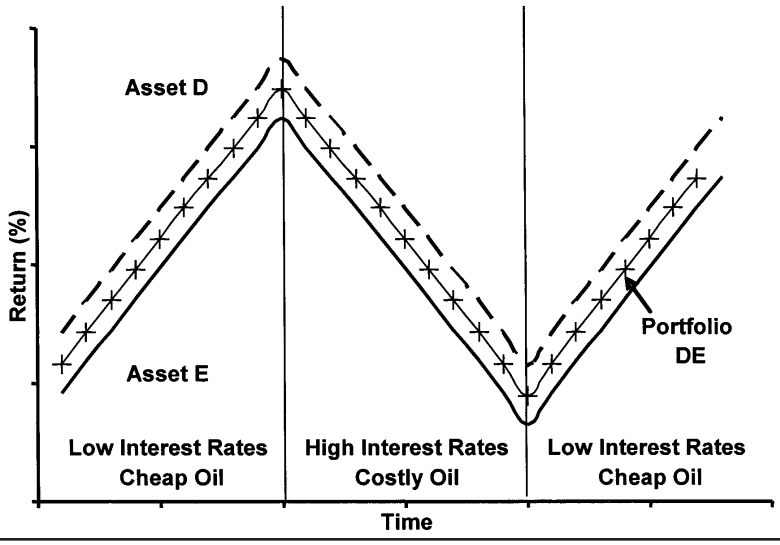


Chart III.2
Return Patterns with Positive Correlation and No Portfolio Effect



E. **The Portfolio Effect Illustrated with Negative Correlation.** To illustrate the portfolio effect, consider two assets, B and R. For exposition assume that Asset B is the common stock of a company that manufactures black ink and Asset R is the common stock of a company that manufactures red ink. When the economy is doing well, the black ink company (B) prospers and the red ink company (R) suffers and vice versa. The returns to these two stocks would not be positively correlated—in fact their tendency to move in opposite directions would suggest a negative correlation. For this hypothetical illustration we have assumed the returns over time create a perfect mirror image (a correlation of -1.0). As Chart III.1 shows, combining these two risky assets in equal proportions into a portfolio (Portfolio BR) offsets or diversifies away much of the risk associated with holding either of the two assets in isolation. While we have not yet computed the standard deviation of these assets or of their portfolio, we can get a sense of the relative risks by comparing the volatility of the three lines traced by these assets' returns over time. Assets B and R are clearly volatile, rising and falling over time. Their portfolio, BR (equal proportions of B and R), is not at all volatile. While this example is unrealistic, it graphically illustrates the portfolio effect.

F. **The Portfolio Effect Illustrated with Positive Correlation.** In reality no two real assets are so strongly negatively correlated. More likely the fiduciary will be working with assets that are influenced by common variables like interest rates and oil prices. Chart III.2 illustrates how Portfolio DE—combining equally the highly positively correlated returns of two stocks D and E—provides no diversification benefit from the portfolio effect. For the purposes of this illustration we have assumed these stocks move in lock-step (a correlation of +1.0). In this Chart the path traced by Portfolio DE matches exactly that of both D and E, providing no amelioration of the volatility because there is no diversification and, therefore, no portfolio effect.

G. **A Numerical Illustration of the Portfolio Effect.** A simple numerical example may help to illustrate further the portfolio effect. Assume we have two different stocks, A and B, with expected returns and standard deviations as shown in Chart III.3. Assume further that the correlation between the returns of these two assets is 0.30.[13]

Chart III.3
Expected Returns and Standard Deviations for Two Assets

	Expected Return	Standard Deviation (Expected Risk)
Stock A	4.60%	5.62%
Stock B	7.30%	5.92%
Portfolio AB	5.95%	4.66%

[13] We have chosen a correlation of 0.30 because this is the average correlation between publicly traded common stocks. The returns and risks are taken from real stocks but their identity is immaterial to our example.

If we invest 50 percent of available funds in stock A and 50 percent in stock B, then the expected return of portfolio AB will be 5.95 percent—halfway between the expected returns of the constituent stocks. The risk of the portfolio as measured by the portfolio's standard deviation will be 4.66 percent.[14] In this case the portfolio effect makes portfolio AB less risky than either of the individual stocks, A and B. Note that the correlation is positive (0.30) so that the returns on these two stocks mimicked each other to some extent, although not perfectly. The resulting portfolio risk reduction illustrates the benefit of combining assets with less than perfectly correlated returns into a portfolio. It also illustrates the importance of estimating the correlations between asset returns in assessing the expected risk of a portfolio.

H. **A Graphical Representation of the Portfolio Effect.** If stocks A and B are plotted with expected return on the vertical axis, expected risk on the horizontal axis and portfolio AB is plotted in the same risk/return space, Chart III.4 illustrates further the portfolio effect. Portfolio AB dominates stock A, *i.e.*, portfolio AB is better because it has a lower expected risk and a higher expected return than stock A. The rational, risk averse investor would clearly prefer portfolio AB to stock A. Portfolio AB, however, does not dominate Stock B because Stock B has both a higher expected return and higher expected risk. The choice between Stock B and Portfolio AB depends on the risk tolerance of the portfolio.

I. **All Possible Portfolios.** Stocks A and B could be combined in different proportions than the 50-50 allocation illustrated by portfolio AB. Risk and return statistics for these alternative portfolios would be different if 49 percent were invested in A and 51 percent in B, 48 percent in A and 52 percent in B, etc. Thousands of different portfolios could be created by combining these assets in different proportions. If additional assets are included as potential investments, a larger set of possible portfolios emerges like those shown in Chart III.5.

Chart III.4
Risk and Return of Two Assets and Their Portfolio

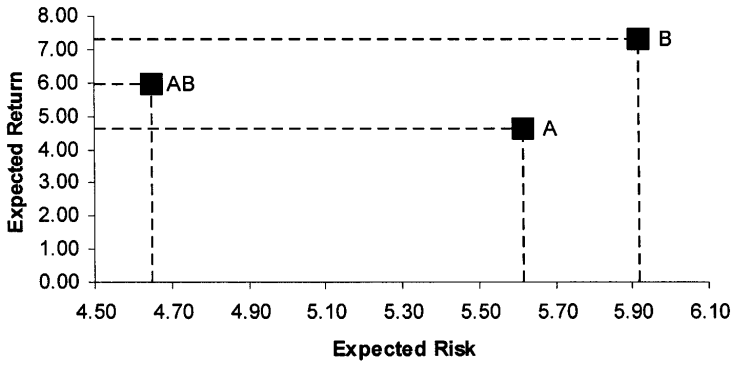
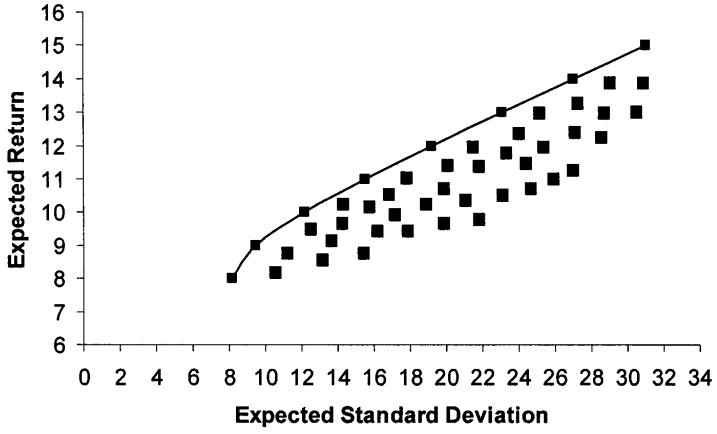


Chart III.5
All Possible Portfolios – Efficient and Non-Efficient



J. **The Efficient Frontier.** All of these portfolios (squares represent different allocations among the assets) are possible in the sense that an investor could invest in them and collectively they comprise what is known as the attainable set of investments. For illustration we have shown only a sample of those possible portfolios. The Efficient Frontier is that subset of possible portfolios connected by the line. The other portfolios are not efficient in that they offer a lower expected return for same the risk as one of the portfolios on the Efficient Frontier (or equivalently, a higher risk for the same expected return possible with one of the efficient portfolios). It follows from the Rule and MPT that a fiduciary should adopt the portfolio that provides the appropriate level of risk and lies on the Efficient Frontier (providing the highest expected return for that level of risk).[15]

K. **Expectations and *a priori* Conduct.** Our discussion thus far has been framed carefully in terms of expectations, respecting the Rule's focus on *a priori* conduct rather than *ex post*

[14] The details of these calculations are discussed in the Appendix.

[15] Up to this point we considered only the mechanical application of MPT. In section IV entitled "A Real World Example," we show how recent advances in MPT imply the fiduciary could exercise judgment in considering portfolios on ***or near*** the efficient frontier.

performance. That is, how the portfolio actually performs (realized returns) is not the standard of judgment —rather it is the fiduciary's formulation of the portfolio that should be evaluated.[16] Once the fiduciary has determined the appropriate level of risk, selected appropriate assets, and forecast their expected returns, risks, and correlations, a reasonable fiduciary should compute the corresponding efficient portfolio. Failure to at least consider efficient portfolios may imply the fiduciary has not acted in the best interest of the beneficiaries. The fiduciary, however, may decide the efficient portfolio is inappropriate after considering conditions of the trust that go beyond the risk and return characteristics of the efficient portfolio. Moreover, as our discussion in section IV suggests, MPT allows the fiduciary some flexibility to apply business judgment such that the prudent fiduciary's actual portfolio may not always be precisely on the Efficient Frontier.

IV. A Real World Example

A. **A Trust's Appropriate Investment Candidates.** Assume a fiduciary has taken responsibility for a trust in the summer of 2004. The fiduciary formulates a portfolio by first determining the risk appropriate to the circumstances of the trust.[17] The fiduciary then should construct an efficient portfolio that promises to deliver the highest return for that level of risk. This fiduciary determines that at least five asset class indices, representing thousands of potential individual investments, would be appropriate under the circumstances: large domestic stocks (the Standard and Poors' 500 Stock Index), corporate and government bonds (Ibbotson Associates' Long-term Corporate and Long-term Government Bond Indices, respectively), real estate (the NAREIT index of real estate investment trusts), and U.S. Treasury bills (constant 30-day maturity). These asset classes would be found in many portfolios. The fiduciary also determines, however, that two other asset classes should be included to fairly represent the appropriate range of assets: small domestic stocks (Ibbotson Associates' Small Stock Index), and foreign stocks (the Morgan Stanley Capital Index of equities domiciled in developed countries in Europe, Asia and the Far East). Collectively these assets are called the "feasible set." MPT guides the fiduciary in constructing an efficient portfolio from these asset classes.

B. **Historical Data and Expectations.** The fiduciary has available the annual historical returns and standard deviations for these seven indices. In the absence of compelling evidence that the future will be materially different than the past (at least over the life of the trust), the fiduciary determines to extrapolate the historical record to develop expected returns, standard deviations, and correlations. The historical record, from the viewpoint of the fiduciary, is presented in Chart IV.1.

Chart IV.1
Annual Historical Returns on Seven Indices
All statistics in %
1972-2003*

	Average Return	Standard Deviation
Small Stocks	17.52	23.47
Foreign Stocks	13.20	22.85
Large Stocks	12.94	17.97
Real Estate	12.19	20.59
Corp Bonds	9.62	11.21
Govt Bonds	9.56	12.11
T-bills	6.35	2.90

*Note: 1972 was chosen due to data availability

C. **Correlations.** Based on risk alone, neither small nor foreign stocks look like good candidates to add to this portfolio even though they have attractive return expectations. A basic tenet of MPT, however, is that correlations and individual assets' risks together determine the risk of a portfolio. Chart IV.2 shows the correlations of the indices in the fiduciary's feasible set of asset classes.

The correlations of returns on small stocks with the other assets range between 0.79 (with real estate) to -0.04 (with T-bills) while the correlations of foreign stocks range between 0.59 (with large stock) and -0.09

Chart IV.2
Historical Correlations
1972-2003

	Small Stocks	Foreign Stocks	Large Stocks	Real Estate	Corp Bonds	Govt Bonds	T-bills
Small Stocks	1.00	0.39	0.66	0.79	0.16	0.05	-0.04
Foreign Stocks	0.39	1.00	0.59	0.25	0.11	0.08	-0.09
Large Stocks	0.66	0.59	1.00	0.52	0.35	0.28	0.03
Real Estate	0.79	0.25	0.52	1.00	0.35	0.25	-0.06
Corp Bonds	0.16	0.11	0.35	0.35	1.00	0.95	-0.01
Govt Bonds	0.05	0.08	0.28	0.25	0.95	1.00	0.03
T-bills	-0.04	-0.09	0.03	-0.06	-0.01	0.03	1.00

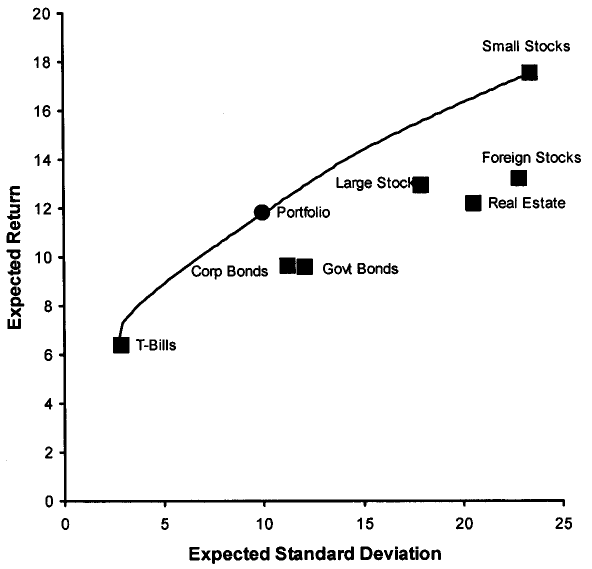
[16] Uniform Prudent Investor Act § 8 and accompanying cmt.

[17] Determining the appropriate risk level is discussed in the second article in this series, "Using a Trust's Investment Policy Statement to Develop the Portfolio's Appropriate Risk Level."

(with T-bills). The fiduciary considers these asset indices appropriate and the portfolio effect suggests that including small and foreign stocks with low correlations with some of the other assets might reduce the portfolio's risk.

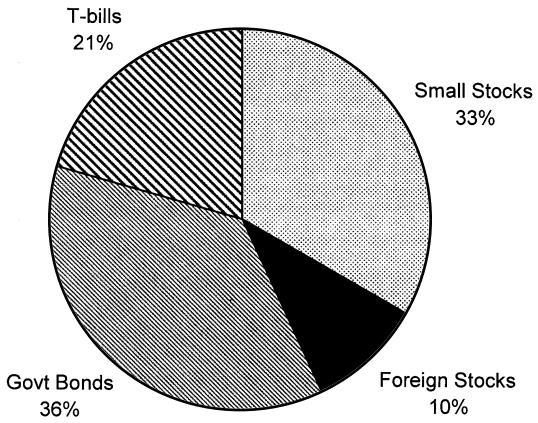
D. **The Efficient Frontier.** The Efficient Frontier shown below was generated using MPT and the historical returns of the seven indices. The line represents the Efficient Frontier and the indices are plotted as squares. The portfolio represented by the round dot is discussed below.

Chart IV.3
Attainable Set of Indices and their Efficient Frontier



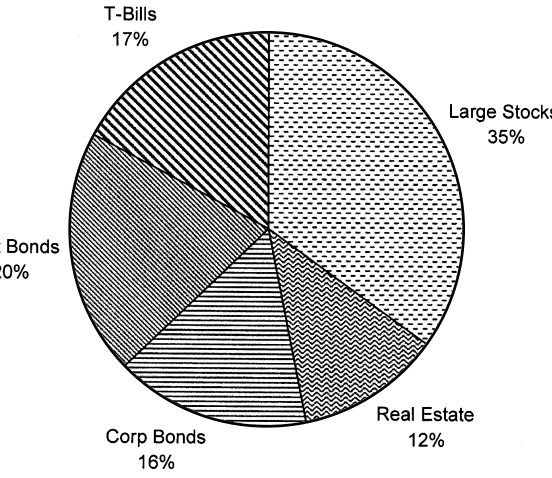
E. **The Efficient Frontier and Individual Portfolios.** The fiduciary's interest is in the efficient portfolios along the Efficient Frontier. For this example we used an investment industry standard mean-variance optimization with expected return, expected risk, and asset correlations for the seven asset classes. This technique computed the portfolio providing the highest (optimal) level of expected return at every risk level, thereby determining all the portfolios on the Efficient Frontier. Note that Small Stocks and T-bills are actually single-asset portfolios on the Efficient Frontier while the other indices (as individual one-asset class portfolios) plot below. This Chart does not, however, reveal the asset allocation of the portfolios along the Efficient Frontier. Not all of these portfolios are relevant as the fiduciary has already chosen a level of risk appropriate to the trust. Assuming for illustrative purposes the standard deviation is 10% per year, the appropriate portfolio is labeled "Portfolio" in Chart IV.3, and its asset allocation is displayed in Chart IV.4.

Chart IV.4
Efficient Portfolio including Small and Foreign Stocks
Annual Expected Return = 11.9%
Expected Standard Deviation = 10%



F. **The Efficient Portfolio Illustrated with Seven Asset Classes.** Chart IV.4 displays the asset allocation of one of the efficient portfolios—the one with an expected annual standard deviation of 10% per year. This portfolio contains both small and foreign stocks, even though they are the riskiest of the indices. Including these indices increases the return on the portfolio, as they are expected to be the two best performing asset classes. This increase in return can be seen by comparing this portfolio with the portfolio displayed in Chart IV.5 constructed by MPT without small and foreign stocks.

Chart IV.5
Efficient Portfolio without Small and Foreign Stocks
Annual Expected Return = 10.5%
Expected Standard Deviation = 10%



G. **The Efficient Portfolio Illustrated with Five Asset Classes.** This portfolio is efficient using the restricted feasible set (excluding small and foreign stocks). The expected return on this portfolio is 1.4%

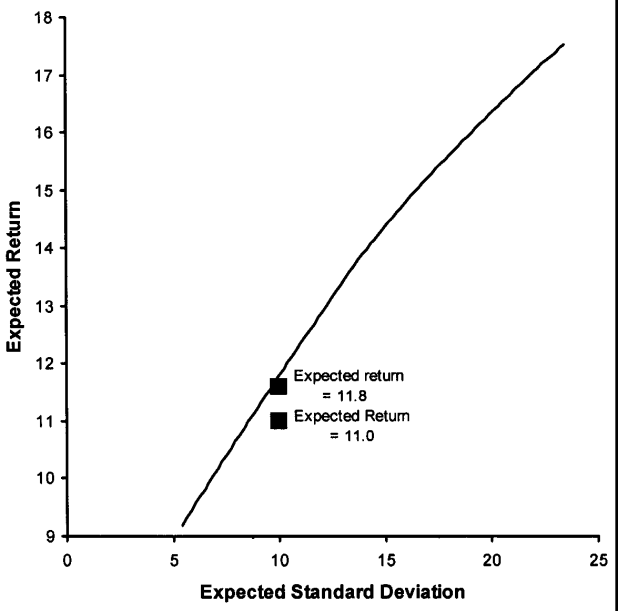
per year less than the portfolio constructed from the full set. Even though the two asset classes, small and foreign stocks, might appear undesirable due to their high risk, they add significantly to the expected return without increasing the expected risk.

H. **The Lesson of MPT Regarding Risky Assets.** This real world example shows that the contribution of an individual asset to the riskiness of a portfolio depends more on its correlations than on its variability. Even though the potential of small and foreign stocks to increase expected return was clear from inspecting the basic data in Chart IV.1, this analysis demonstrates how difficult it is to determine how an asset will affect a portfolio without using MPT. A fiduciary who rejects risky assets without considering their possible role in enhancing portfolio return is inconsistent with the Rule and MPT.

I. **Efficient Portfolios Not on the Efficient Frontier.** Few experts would insist that the fiduciary's portfolio always be on the Efficient Frontier. These experts recognize that portfolios on the Efficient Frontier are optimal, in the sense they offer the highest return for a given level of risk, but they may not make business sense. The Rule implies fiduciaries should look beyond the mechanical rules of MPT and exercise sound business judgment in managing investments. In this spirit the theory of MPT has been expanded to incorporate portfolios that lie below the Efficient Frontier but might, under the circumstances, be more reasonable. The efficient portfolio shown above is a good example. Although the fiduciary started with a set of at least five, and possibly seven, asset classes, the efficient portfolio in Chart IV.4 contains just four asset classes and only two of those (Government bonds and Treasury Bills) were in the original set. Recent advances in MPT have expanded the view of the Efficient Frontier to be a band, rather than a narrow set. Without delving into the details or the calculations, the intuitive appeal of these approaches is they consider portfolios that are below the line to be efficient.[18] How far below the line is a matter of judgment. In the case we have used here as an example the fiduciary might want to explore portfolios that have the required risk level (10%) and slightly lower expected returns. Chart IV.6 shows two of these alternative portfolios—both with an expected standard deviation of 10% but expected returns of 11.8% and 11.0%, respectively.

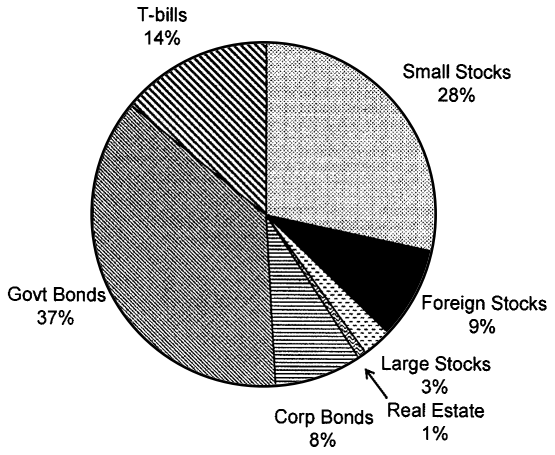
Compared to the efficient portfolio with all seven asset classes (Chart IV.4), the alternative nearly efficient portfolios in Chart IV.6 (displayed in Charts IV.7 and IV.8) could be considered because they involve more asset classes at a minimal cost in percent return per year.

Chart IV.6
Two Nearly Efficient Portfolios
With Expected Standard Deviation = 10%
Below the Efficient Frontier

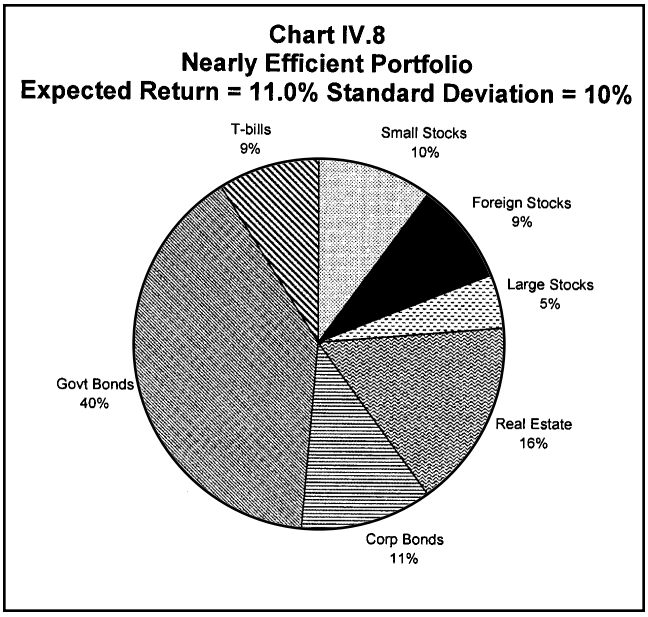


J. **Strict MPT Efficiency and Diversification.** Because the fiduciary had determined that at a minimum the first five asset classes (large stocks, real estate, government and corporate bonds, and Trea-

Chart IV.7
Nearly Efficient Portfolio
Expected Return = 11.8% Standard Deviation = 10%



[18] For further information on these approaches consult Harry M. Markowitz and Nilufer Usmen, "Resampled Frontiers versus Diffuse Bayes: An Experiment," *Journal of Investment Management,* vol. 1, no. 4 (Fourth Quarter 2003), p. 9-25.



sury Bills) should be used in the portfolio and considered both small and foreign stocks as possible additions, either of these portfolios (Chart IV.7 or IV.8) could be considered prudent. Again, how much return below that offered by the strictly efficient portfolio (recall in our example the portfolio on the frontier had an expected return of 11.9%) the fiduciary should accept and still be prudent is a matter of judgment and should be justified. Extensions of MPT like those illustrated in Charts IV.7 and IV.8 allow the fiduciary to accommodate the Rule's requirement for diversification within the framework of MPT's efficient portfolios. In our example a reasonable judgment could be that these nearly efficient portfolios require a small sacrifice in exchange for broader diversification. If the fiduciary selects a nearly efficient portfolio, supporting documentation should include an analysis similar to the one shown here in order to assess the diminution of expected return for that given risk level.

K. **Assessing Potential Damages.** If the fiduciary has determined an appropriate risk level for the portfolio through the careful development of an Investment Policy Statement,[19] then the fiduciary's conduct can be assessed by the distance between the actual portfolio and an efficient or nearly efficient portfolio at that risk level. Assessing conduct (and potential damages) will be covered in greater detail in the third installment of this series.

V. Conclusions

A. **The Prudent Investor Rule and Modern Portfolio Theory.** The Prudent Investor Rule incorporates Modern Portfolio Theory. Fiduciaries are thus bound to consider MPT in constructing the portfolio under their control. To do otherwise exposes the fiduciary to claims of misconduct and the resulting potential assessment of damages and surcharges.

B. **Fiduciary Conduct under MPT and the Rule.** The Rule is a test of conduct, not performance.[20] Thus, if the fiduciary chooses a risk level that is appropriate under the terms of the trust and constructs a portfolio that is ex ante efficient and the resulting portfolio suffers losses, the fiduciary should not be liable for damages. To use MPT the fiduciary must select assets that fairly represent the feasible set and develop reasonable expectations for return, risk, and correlations. The fiduciary should then construct the efficient portfolio for the chosen risk level. If the fiduciary then decides, either because that portfolio is inconsistent with the terms of the trust or because a more diversified portfolio is available with a minor sacrifice in return, not to implement that portfolio, that decision must be justified. The fiduciary should use sound judgment in making portfolio allocation decisions. While there is no explicit requirement that the fiduciary use the mechanics of MPT, constructing a portfolio using business judgment alone is difficult to explain and support satisfactorily. A more defensible practice would be for the fiduciary to begin with the portfolio recommended by MPT. The tools to develop these portfolios are readily available to investment practitioners and should pose no barrier to the conscientious fiduciary. To ignore these tools places the fiduciary in a precarious position.

VI. Appendix: Expected Return, Variance, Standard Deviation, and Correlation Calculations for Stocks A and B

For those inclined mathematically and desiring to understand the basis of calculations used, the following information is provided. Chart A.1 illustrates hypothetical annual return data from 1994 through 2003 for stocks A (column 1) and B (column 2). Data for an equally weighted portfolio of these stocks is shown in column (3). While widely available software, like Microsoft's Excel, make calculating the required statistics simple, this Appendix shows some of the details behind those calculations for our two asset example.

[19] Investment Policy Statements are discussed in the second article in the series, "Using a Trust's Investment Policy Statement to Develop the Portfolio's Appropriate Risk Level."

[20] See *supra* text accompanying note 16.

Chart A.1
Historical Data

Year	(1) Asset A Return	(2) Asset B Return	(3) Portfolio AB Return*
1994	16.0	12.0	14.0
1995	5.0	-2.0	1.5
1996	5.0	10.0	7.5
1997	6.0	6.0	6.0
1998	-4.0	11.0	3.5
1999	6.0	3.0	4.5
2000	0.0	-1.0	-0.5
2001	3.0	16.0	9.5
2002	-2.0	4.0	1.0
2003	11.0	14.0	12.5

* 50% invested in Asset A and 50% in Asset B

A. **Symbols. This Appendix uses the following symbols:**

$\overline{A}$ = average returns for stock A, $\overline{B}$ = average returns for stock B and $\overline{AB}$ = average returns for portfolio AB.

$\sum_{t=1}^{10}(\cdot)$ = add the terms following the symbol (*e.g.*, in the parentheses) denoted by the time subscript t. Our example covers ten years of data so ten terms will be added.

σ_i^2 = variance of stock or portfolio where i indicates the asset

σ_i = standard deviation of stock or portfolio where i indicates the asset

$\sqrt{\ }$ = square root function

$\rho_{i,j}$ = correlation between stocks i and j

w_i = weight assigned to asset i in the portfolio

Note: In a portfolio the weights sum to 1 (100%) indicating the portfolio is fully invested.

B. **Average Returns.** The average returns for Stocks A and B and their portfolio are calculated as follows:

$$\overline{A} = \frac{1}{10}\sum_{t=1}^{10} A_t = \frac{16+5+5+6-4+6+0+3-2+11}{10} = 4.60\%$$

where $\overline{A}$ represents the average for stock A and the ten A_t are annual returns. This calculation is the same as the simple average familiar to most people.

Similarly:

$$\overline{B} = \frac{1}{10}\sum_{t=1}^{10} B_t = 7.30\% \text{ and } \overline{AB} = \frac{1}{10}\sum_{t=1}^{10} AB_t = 5.95\%$$

Because the portfolio's average return is the weighted sum of the average returns of the assets in the portfolio and the weights in this example are 50 percent invested in each stock, the portfolio's average return is also:

$\overline{AB} = .5 * \overline{A} + .5 * \overline{B} = 0.5*4.60\% + 0.5*7.30\% = 5.95\%,$ the same result as before.

C. **Calculating the Risk of Individual Stocks.** The first step in calculating risk of individual stocks is to compute their variance – the average squared difference between the periodic returns and their averages as follows:

$$\sigma_A^2 = \frac{1}{10}\sum_{t=1}^{10}(A_t - \overline{A})^2 = 31.58\% \text{ and } \sigma_B^2 = \frac{1}{10}\sum_{t=1}^{10}(B_t - \overline{B})^2 = 35.05\%$$

Because risk is measured in MPT as the standard deviation and the standard deviation is the square root of the variance, the risks (standard deviation of returns) for stocks A and B are as follows:

$$\sigma_A = \sqrt{\sigma_A^2} = \sqrt{31.58\%} = 5.62\% \text{ and } \sigma_B = \sqrt{\sigma_B^2} = \sqrt{35.05\%} = 5.92\%$$

These are the statistics reported in Chart III.3.

D. **Calculating the Risk of the Portfolio**

1. Correlation. To compute the portfolio's standard deviation from the statistics for its constituent assets the correlation must first be computed:

$$\rho = \frac{\frac{1}{10}\sum_{t=1}^{10}(A_t - \overline{A})(B_t - \overline{B})}{\sigma_A \sigma_B} = 0.30$$

This equation demonstrates that the correlation is the product of the average difference between the contemporaneous returns, relative to the product of their standard deviations.

2. Standard Deviation. The standard deviation for any two asset portfolio can be calculated using the following general equation:

$$\sigma_{AB} = \sqrt{w_A^2\sigma_A^2 + w_B^2\sigma_B^2 + 2w_A w_B \rho_{AB}}\,\sigma_A\sigma_B$$

In this case:

$$\sigma_{AB} = \sqrt{(0.5)^2\sigma_A^2 + (0.5)^2\sigma_B^2 + 2(0.5)(0.5)\rho_{AB}}\,\sigma_A\sigma_B =$$

$$\sqrt{.25*31.58 + .25*35.05 + 2*.25*0.30*5.62*5.92} = 4.66\%$$

E. **Illustrating the Portfolio Effect.** The portfolio effect can now be illustrated using the general equation for the standard deviation of a two asset portfolio shown above. The first two terms, $w_A^2\sigma_A^2 + w_B^2\sigma_B^2$, combine the variances of the two assets. The last term $(2w_A w_B \rho \sigma_A \sigma_B)$ measures the portfolio effect. Mathematically, the lower the correlation, ρ, the lower the overall risk of the portfolio. This relationship can be generalized to hold for portfolios with many assets as well. Moreover, as the number of assets increases, the individual weights, (w_i) which are all less than one, will become progressively smaller numbers and their squares, w_i^2, smaller still. The last term also becomes smaller but only as the product of the weights, not their squares. Increasing the number of assets enhances the portfolio effect as most of the standard deviation will come from the last term.